PHYS 101: General Physics 1 **KOÇ UNIVERSITY** **Fall Semestre 2010**

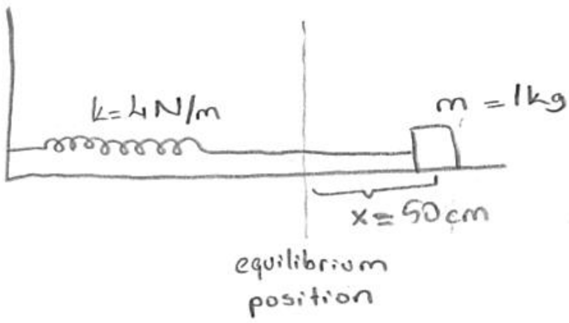
College of Sciences

Section 1 **Quiz 9** **16 December 2010**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

A body of mass $m = 1$ kg is attached to a wall by a spring of constant $k = 4$ N/m. It is initially pulled a distance $x = 50$ cm from its equilibrium position and released from rest. (i) What are the amplitude, frequency and period of the motion? (ii) Find the displacement and the velocity for time $t = 15$ s after release.



i) $x(t) = A.\cos\omega t$

in order to find A, we need to apply the initial conditions:

$\begin{cases} x(0) = 50 \text{ cm} \\ v(0) = 0 \end{cases}$

$x(0) = A = 50\text{cm} = 0{,}5\text{m}$

$\frac{dx(t)}{dt} = v(t) = -A\omega.\sin\omega t$

$v(0) = 0$ ✓

$T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{k/m}} = \frac{2\pi}{\sqrt{4/1}} = \boxed{\pi = T}$

$\boxed{f = \frac{1}{T} = \frac{1}{\pi}}$

ii) $x(t) = A.\cos\omega t = 0{,}5\cos 2t$ (A → 0,5; ω → 2)

$x(t=15) = 0{,}5.\cos 30 = \frac{\sqrt{3}}{4}$ m.

$v(t) = -A\omega\sin\omega t = -0{,}5.2.\sin 2t$

$v(t=15) = -\sin 30 = -\frac{1}{2}$ m/s.

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Section 2 **Quiz 9** **16 December 2010**

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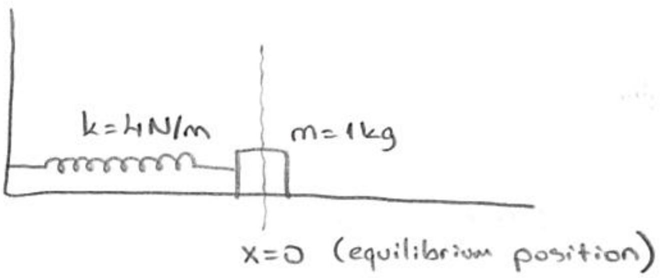
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

A body of mass $m = 1$ kg is attached to a wall by a spring of constant $k = 4$ N/m. It is given an initial velocity, at $x = 0$ cm (its equilibrium position), of 6 m/s. Plot the time evolution of the displacement $x(t)$, velocity $v(t)$ and acceleration $a(t)$ for one period.



The function of $x(t)$ is either $A.\cos\omega t$ or $A.\sin\omega t$. We need to try both functions with initial conditions:

$$\begin{cases} x(0) = 0 \\ v(0) = 6\,m/s \end{cases}$$

$x(t) = A\cos\omega t$

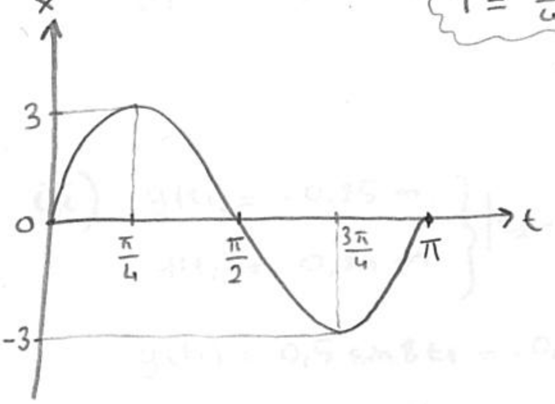
$t=0 \Rightarrow x(0) = A \neq 0$, so $x(t) \neq A\cos\omega t$

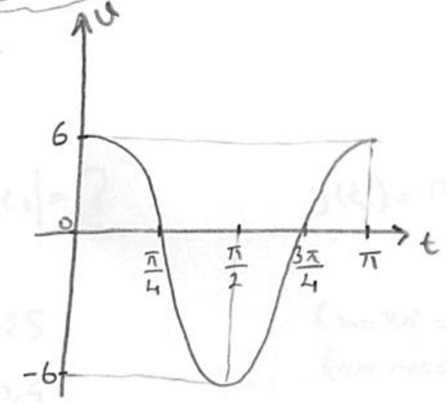
$\boxed{x(t) = A\sin\omega t}$ $\quad$ $\boxed{v(t) = \frac{dx}{dt} = A\omega.\cos\omega t}$

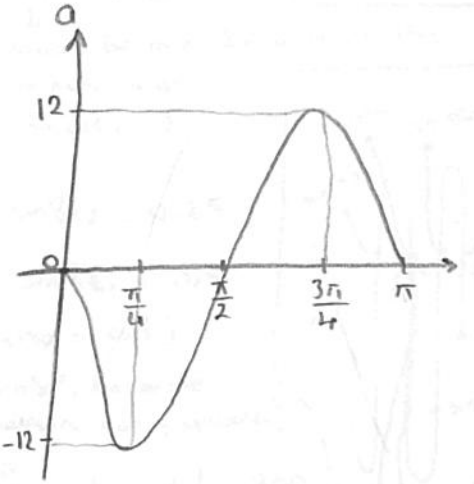
$t=0 \Rightarrow x(0) = 0$ ✓ $\quad$ $t=0 \Rightarrow v(0) = A\omega = 6$ ✓

$\boxed{a(t) = \frac{dv}{dt} = -A\omega^2\sin\omega t}$ $\quad$ $\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{4}{1}} = 2$ $\quad$ so $\boxed{A = 3}$ $\boxed{\omega = 2}$

$T = \frac{2\pi}{\omega} = \pi$







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Section 3 **Quiz 9** **16 December 2010**

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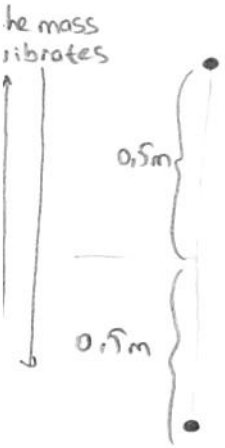
Quiz duration: 10 minutes

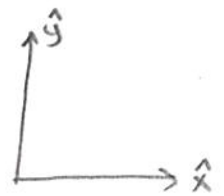
Name: **Student ID:** **Signature:**

A 2 kg mass vibrates in simple harmonic motion with amplitude 50 cm and angular frequency 8 rad/s. Find (i) the energy of motion (ii) the maximum values of the velocity and the acceleration (iii) the time it takes to move from 25 cm below to 25 cm above the equilibrium position.



$m = 2kg$ $\omega = 8$ rad/s
$A = 0,5$ m

x=0 (equilibrium position)



i) $E = \frac{1}{2} m v^2 + mgh$

(h is the height from the position $y = -0,5m$)

$E = \frac{1}{2} m (A\omega \cos\omega t)^2 + mgh$

ii) $y(t) = A . \sin\omega t = 0,5 \sin 8t$

$\frac{dy}{dt} = v_y(t) = A\omega \cos\omega t = 4 . \cos 8t \Rightarrow v_{max} = A\omega \cos\omega t = A\omega = \boxed{4 \text{ m/s} = v_{max}}$

should be max. $\cos\omega t = 1$

$\frac{dv}{dt} = a_y(t) = -A\omega^2 \sin\omega t = -32 \sin 8t \Rightarrow a_{max} = -A\omega^2 \sin\omega t = A\omega^2 = \boxed{32 \text{ m/s}^2 = a_{max}}$

should be min. because of the minus sign. $\sin\omega t = -1$

iii) $y(t_1) = -0,25$ m, $y(t_2) = 0,25$ m $\Big\} |t_2 - t_1| = ?$

$y(t_1) = 0,5 \sin 8t_1 = -0,25$

$\sin 8t_1 = -0,5$

($\sin 330° = \sin 210° = -0,5$)

$8t_1 = 330$

(we need $\sin 330°$, because at $\sin 210°$ mass is going downwards.) $\boxed{t_1 = \frac{330}{8} s.}$

$y(t_2) = 0,5 \sin 8t_2 = 0,25$

$\sin 8t_2 = 0,5$

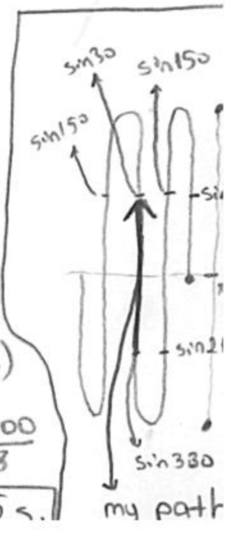
($\sin 30° = \sin 150° = 0,5$)

(we need $\sin 30°$, because at $\sin 30$ mass is going upwords)

$8t_2 = 30$

$\boxed{t_2 = \frac{30}{8}}$ $|t_2 - t_1| = \frac{300}{8}$

$\boxed{\Delta t = 37,5 s.}$



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Section 4 **Quiz 9** **16 December 2010**

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Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

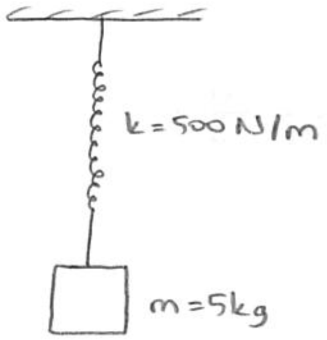
A spring stretches 10 cm when its tension is 50 N. A body of mass 5 kg is hung from the spring. When at rest the body is given an initial upward velocity of 1 m/s. (Take $g = 10 \text{m/s}^2$) (i) Find the amplitude and the frequency of the motion. (ii) Find the acceleration of the mass when it is 5 cm above its equilibrium position. (iii) Find the force of tension in the spring at this point.

$\vec{F} = -k\vec{x}$

$|\vec{F}| = |-k\vec{x}|$

$50 = k \cdot 0,1$

$\boxed{k = 500 \text{ N/m}}$



$\left\{ \begin{array}{l} x(0) = 0 \\ v(0) = 1 \text{ m/s} \end{array} \right\}$

$\omega = \sqrt{\frac{k}{m}} = \sqrt{\frac{500}{5}} = 10 \text{rad/}$

i) $x(t) = A . \sin\omega t$

$t=0 \Rightarrow x(0) = 0$ ✓

$v(t) = \frac{dx}{dt} = A\omega \cos\omega t$

$v(0) = 1 = A . 10 \Rightarrow \boxed{A = 0,1 \text{ m}}$

$f = \frac{\omega}{2\pi} = \frac{10}{2\pi} = \boxed{\frac{5}{\pi} = f}$

ii) $a(t) = \frac{dv}{dt} = -A\omega^2 \sin\omega t = -10 \sin 10t$

$x(t_1) = 0,05 = 0,1 \sin 10 t_1$

$\sin 10t_1 = 0,5 = \sin 30$

$10t_1 = 30$

$\boxed{t_1 = 3s.}$

$a(t_1 = 3) = -10 \sin 30 = -5 \text{ m/s}^2$

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iii) $x(t_1) = 0,05$

$F = kx = 500 \cdot 0,05 = 25\text{N}$

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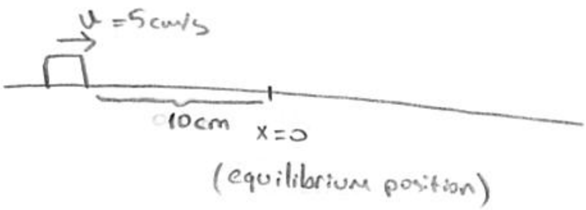
Section 5 **Quiz 9** **16 December 2010**

Closed book. No calculators are to be used for this quiz.
Quiz duration: 10 minutes

Name: **Student ID:** **Signature:**

A body in simple harmonic motion with angular frequency $\omega = 0.5$ rad/s is initially 10 cm away from its equilibrium position and moving back toward the equilibrium position with a velocity 5 cm/s. (i) Find the period of the motion. (ii) Find the coordinate and velocity of the body as a function of time. (iii) How long does it take the body to return to its equilibrium position?



$\omega = 0{,}5$ rad/s

i) $T = \frac{2\pi}{\omega} = 4\pi$

ii) $x = A.\sin(\omega t + \phi)$

$t=0 \Rightarrow x(0) = A.\sin\phi = 10\text{ cm}$

$v(t) = \frac{dx}{dt} = A\omega\cos(\omega t + \phi)$

$t=0 \Rightarrow v(0) = A\omega\cos\phi = 5\text{ cm/s}$

$A.\sin 45^\circ = 10$

$A.\frac{\sqrt{2}}{2} = 10$

$\boxed{A = 10\sqrt{2}\text{ cm}}$

$\frac{A\sin\phi = 10}{\frac{A}{2}\cos\phi = 5} \Rightarrow \tan\phi = 1$

$\phi = 45^\circ = \frac{\pi}{4}$

$\boxed{x(t) = 10\sqrt{2}.\sin\left(\frac{t}{2} + 45^\circ\right)}$

$\boxed{v(t) = 5\sqrt{2}.\cos\left(\frac{t}{2} + 45^\circ\right)}$

iii) $x(0) = 10\text{ cm}$

$x(t_1) = 0 = 10\sqrt{2}\sin\left(\frac{t_1}{2} + 45\right) = 10\sqrt{2}\sin\left(\frac{t_1}{2} + \frac{\pi}{4}\right)$

$0, \pi, 2\pi, \ldots$

$\frac{t_1}{2} + \frac{\pi}{4} = 0$

$t_1 = -\frac{\pi}{2}$

we cannot take $t_1 < 0$

$\frac{t_1}{2} + \frac{\pi}{4} = \pi$

$\boxed{t_1 = \frac{3\pi}{2}\text{ s.}}$